



An Overview on Fuzzy Logic & Fuzzy Elements

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Abstract: Fuzzy logic is used in the applications which involves vagueness, hesitation, linguistic uncertainty, measurement imprecision, natural diversity and subjectivity . In this paper ,we study the different approaches such as Trapezoidal , Triangular fuzzifiers used for conversion of crisp variables to linguistic variables, different fuzzy inference approaches such as Mamdani, Takagi-Sugeno and defuzzification approaches such as center of gravity, mean of maximum are also covered.

Key words: membership function ,fuzzification, defuzzification,Mamdani,Takagi-Sugeno

I. Introduction

Fuzzy ideas and fuzzy logic are so often utilized in our routine life that nobody even pays attention to them. For instance, to answer some questions in certain surveys, most time one could answer with 'Not Very Satisfied' or 'Quite Satisfied', which are also fuzzy or ambiguous answers. Exactly to what degree is one satisfied or dissatisfied with some service or product for those surveys? These vague answers can only be created and implemented by human beings, but not machines.

Is it possible for a computer to answer those survey questions directly as a human beings did? It is absolutely impossible. Computers can only understand either '0' or '1', and 'HIGH' or 'LOW'. Those data are called crisp or classic data and can be processed by all machines. Is it possible to allow computers to handle those ambiguous data with the help of a human being? If so, how can computers and machines handle those vague data? The answer to the first question is yes. But to answer the second question ,we need some fuzzy logic techniques and knowledge of fuzzy inference system[1].

II. Why Fuzzy Logic ?

Classical logic assumes that each sentence is either true or false. But this standard creates a problem when describing ambiguous, inexact phenomena and formalizing the intermediate situations. For example, let us consider RBC – the index which means the number of red blood cells in blood morphology. How can it be determined whether the value of RBC, which is 6.4 mln/mm^3 , is high? If the person who has been tested is an adult woman then this situation is evident – the value of RBC is high. For a 30-year-old man who every day does hard physical work it is a value classified as standard. By contrast, if the patient is an infant, it arises doubts. Then it is a problem to say what we think about it. It would be the safest to use an expression that this ratio is rather below standard. Interpretation is dependent on the specific situation and its context. First attempt to resolve it was taken by the Polish scientist Jan Łukasiewicz, who formed three-valued logic. In this system a value is formulated “possible” between true and false. However, the real breakthrough was the work of Lotfi A. Zadeh entitled “Fuzzy sets” published in 1965. Here it was defined that fuzzy sets differ from classical approach, which assume that an item belongs to the set or not. They do not have sharp, clearly defined border. Each element belongs to the fuzzy set to a certain extent and their attachment may be expressed as a number in the range [0,1]. Such classification is like the human process of thinking, reasoning and interpreting occurrence. It allows an individual approach to each circumstance as well as formalized situation described in a natural language. So Lotfi A. Zadeh proposed in 1973 a fuzzy logic. Some statements in this system may be false (0), true (1) or in some part true[2].

III. Fuzzy Elements

A fuzzy set is a set with fuzzy boundaries. Defined fuzzy sets or classes for each variable allows intermediate grades of membership in them, which means each set could have elements that belongs partially to it; the degree of belonging is called membership functions ranging from 0 to 1. Fuzzy sets are found whenever there is some ambiguity or subjectivity, and thus imprecisely worded conditions to belong to the set.

1.Membership Function

Degree of belonging of item x to set A specifies membership function which will be recorded as $\mu_A(x)$. Its shape or model may be decided by expert knowledge, or a neural network.

Fuzzy logic is a set of mathematical principles for knowledge representation based on degrees of membership rather than on crisp membership of classical binary logic.

A classical set is a collection of objects in a given range with a sharp boundary. An object can either belong to the set or not belong to the set. For example, we assume to create a faculty set or a faculty collection A with ten-faculty members x_1, x_2, \dots, x_{10} , in a college:

$$A = \{ x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} \}$$

In general, the entire object of discussion A is called a universe of discourse, and each member x_i is called an element. The fuzzy set is represented by a membership function, defined as follows:

$$\mu_A: X \rightarrow [0,1]$$

$$\mu_A(X) = 1 \text{ if } X \text{ is totally in } A$$

$$\mu_A(X) = 0 \text{ if } X \text{ is not in } A$$

$$0 < \mu_A(X) < 1 \text{ if } X \text{ is partially in } A$$

$\mu_{A(x)}$ expresses to which the value x belongs to the fuzzy set A . The value 0 corresponds to the absolute non-membership and the value 1 corresponds to the absolute membership. Therefore, a fuzzy membership function indicates the degree of belonging of some element x of the universe of discourse A . It maps each element of X to a membership grade between 0 and 1 in various shapes such as Triangular, Trapezoidal, Sigmoid and Gaussian[4].

Let us consider the problem presented earlier, "Is the value of RBC equal 6.4 mln/mm^3 high?". Then membership function may take the form as in Figure 3.1

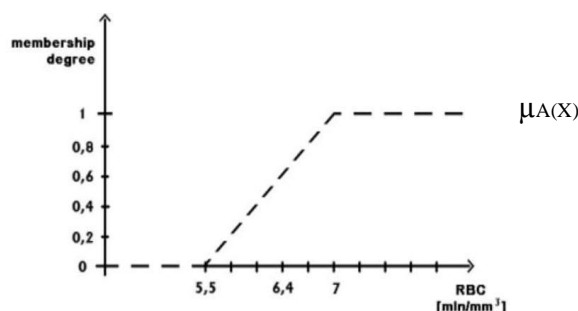
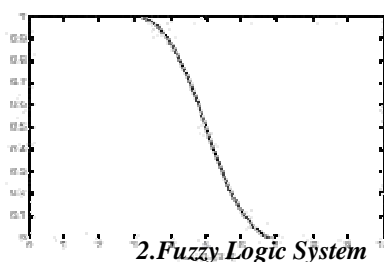


Fig. 3.1 The membership function of measurement results RBC belonging to a set of indicators of high or like that

Thus, in this interpretation, the number of red blood cells of 6,4 as being 0.6 high value is defined. Furthermore, the indicator which is 5.5 mln/mm^3 , does not belong to the set of high values because membership degree is 0. On the other hand, each of the results of over 7 mln/mm^3 is blindly recognized for the high – membership degree is 1. The membership function gives two fold information. It designates areas where there is no doubt with assigning certain element to a particular set but also shows how to define the degree of fulfilled criteria of belonging to the set in the interval in which there is some confusion[2].



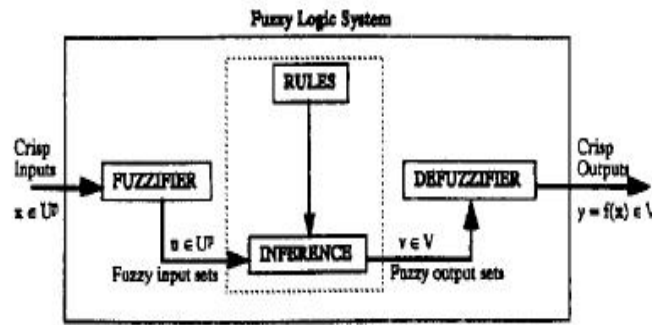
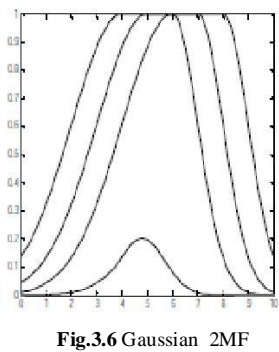
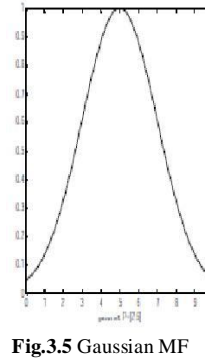
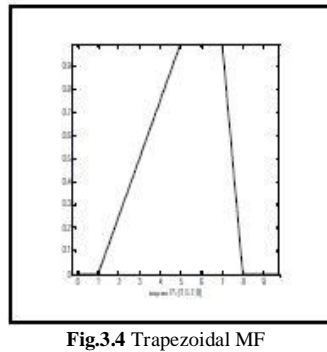
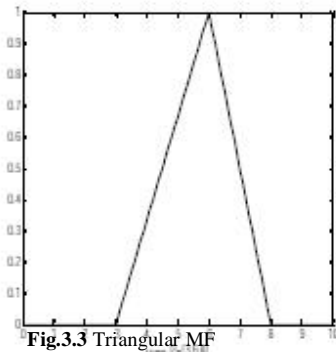


Fig. 3.2:Fuzzy Logic System

Fuzzy logic opens up the possibility for users to formalize the rules of everyday life through the fuzzy inference. This operation consists of a series of stages that can be carried out in many different ways. There are processes such as defuzzification (blurring), inference, fuzzification (sharpening) are explained below. Furthermore, numerous fuzzy models were created to increase the accuracy of fuzzy reasoning or its simplification as, for example such models as: Mamdani, Takagi-Sugeno, relational, local, global, multimodels[5].

a.Fuzzification – convert classical data or crisp data into fuzzy data or Membership Functions (MFs). It involves two processes: derive the membership functions for input and output variables and represent them with linguistic variables. It is achieved with different types of fuzzifiers. There are generally eleven types of fuzzifiers, which are used for the fuzzification process. They are: Trapezoidal, Triangular, Singleton, Gaussian, bell shaped, Sigmoidal, S-curve fuzzifier. The exact type depends on the actual applications. For those systems that need significant dynamic variation in a short period of time, a triangular or trapezoidal waveform should be utilized. For those system that need very high control accuracy, a Gaussian or S-curve waveform should be selected.

The Fuzzy Logic includes 11 built-in membership function types. These 11 functions are, in turn, built from several basic functions: piecewise linear functions, the Gaussian distribution function, the sigmoid curve, and quadratic and cubic polynomial curves. The simplest membership functions are formed using straight lines. Of these, the simplest is the triangular membership function, and it has the function name trimf. It is nothing more than a collection of three points forming a triangle. The graphical representation of triangular and trapezoidal membership functions are given in Fig.3.3 and Fig 3.4[7].



Two membership functions gaussmf & gauss2mf are built on the Gaussian distribution curve are illustrated in the Fig 3.5 & Fig 3.6 respectively. The generalized bell membership function (Fig 3.7) is specified by three parameters and has the function name gbellmf. The bell membership function has one more parameter than the Gaussian membership function.

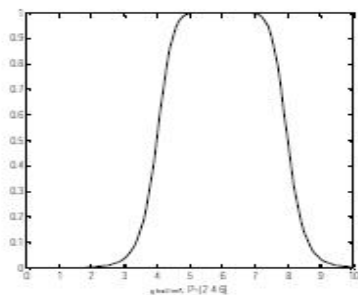


Fig.3.7 Gbell MF

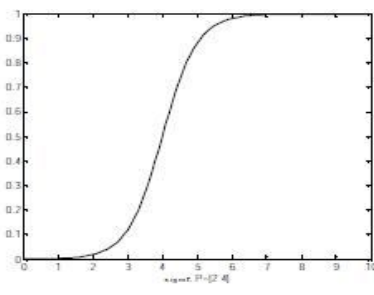


Fig.3.8 Sigmoidal MF

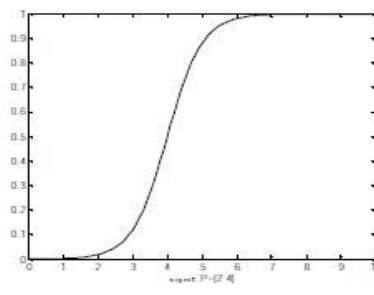


Fig.3.9 Z-shaped MF

Although the Gaussian membership functions and bell membership functions achieve smoothness, they are unable to specify asymmetric membership functions, which are important in certain applications. Next we define the sigmoidal membership function (Fig 3.8), which is either open left or right.

Polynomial based curves account for several of the membership functions in the toolbox. Three related membership functions are the Z, S, and Pi curves, all named because of their shape. The function zmf (Fig 3.9) is the asymmetrical polynomial curve open to the left.

If the task would be to qualify the results of red blood cells measurement to one of three groups: high, normal or low, the membership function as presented in Figure 3.10 may be used.

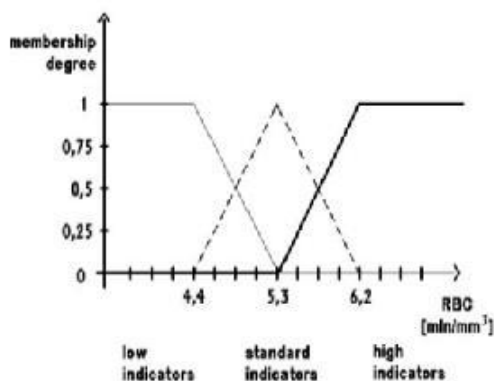


Fig. 3.10. Membership functions to sets of different types of RBC measurement

Above figure shows, the value of rate which is $5,525 \text{ mln/mm}^3$ belongs to high indicators in 0,25 and at the same time it belongs to the set of standard indicators in 0,75. On the other hand, it isn't contained in the set of low indicators as evidenced by the zero membership degree.

b.Fuzzy Inference Process – The process of drawing conclusion from existing data is called inference. Fuzzy Inference Process combine membership functions with the control rules to derive the fuzzy output. The fuzzy inference engine uses the rules in the knowledge-base and derives conclusion base on the rules.

By using linguistic variable, fuzzy if-then rules would be set up: generally presented in the form of: if x is A then y is B where x and y are linguistic variables and A and B are linguistic values, determined by their fuzzy sets. The first part of the rule is called the antecedent, and can consist of multiple parts with the operators AND or OR between them. The latter part is called the consequent, and can also include several outputs.

i) Mamdani-type fuzzy rule:

The default type of rule generally used is the Mamdani type. In Mamdani approach, we don't have any logical combination of inputs with AND/OR because antecedent part of all rules has one section.

- If x is small Then y is large
- If x is medium Then y is medium
- If x is large Then y is small.

ii) TSK-type fuzzy rule:

If X1 is A1 and ... and Xn is An then Y = p0 + p1X1 + ... + pnXn

- If x is small Then y=2x
- If x is medium Then y=-x + 3
- If x is large Then y=x-1

c. Defuzzification –The defuzzification process translates the output from the inference engine into crisp output.

The input to the defuzzification process is a fuzzy set while the output of the defuzzification process is a single number (crisp output). Many defuzzification techniques are proposed and four common defuzzification techniques are: center-of-area (gravity), center-of-sums, mean of maxima.

i) Mean of Maxima(MOM) Method

The Mean of Maxima (MOM) defuzzification method computes the average of those fuzzy conclusions or outputs that have the highest degrees[3].

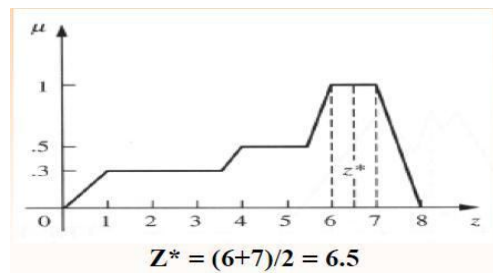


Fig. 3.11 (a) MOM method example

ii) The Center of Gravity method (COG)

The center-of-area (also referred as center-of-gravity or the centroid method) is the most widely used technique because when it is used, the defuzzified values tend to move smoothly around the fuzzy output region, thus giving a more accurate representation of fuzzy set of any shape . The center-of-gravity (CoG) often uses discrete variables so that CoG, Y' can be approximated to overcome its disadvantage as shown in equation below which uses weighted average of the centers of the fuzzy set instead of integration. The CoG is an averaging technique. The CoG defuzzification method is similar to the formula for calculating the center of gravity in physics. The difference is that, density of mass is replaced by the membership values. The CoG formula is given as:

$$COG(Y') = \frac{\sum \mu_Y(X_i) X_i}{\sum \mu_Y(X_i)}$$

Where $\mu_Y(X_i)$ = Membership value in the membership function and X_i = center of membership function.
A graphic representation of the COG method is shown in Figure below.

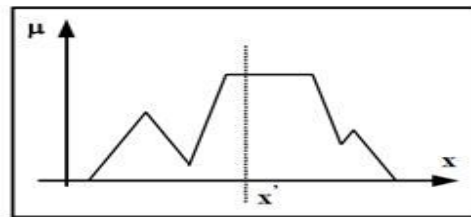


Fig. 3.11 (b) COG method example

IV. Fuzzy Applications

1. They are used in refrigeration, air-conditioning equipment and car driving control systems, water treatment devices, in elevators, cameras with auto focus and household appliances like kitchen dishwashers, washing machines, refrigerators.

2. In decision-making processes relating to trading activities of enterprises, to credit risk assessment, in systems controlling the rolling stock or ventilation of underground tunnels, to solve the problem of traffic jams, and even in the production of Japanese sake alcohol.

3. Fuzzy logic has numerous medical applications, among others, in fields such as cardiology, oncology, endocrinology, pediatrics, intensive care, anesthesiology. It supports the processes of making diagnoses or determining the dose of medicine. It participates in decisions concerning treatment and arising from a number of factors, it may be used also to predict patient length of stay in hospital.

V. Conclusion

A review of the fuzzy sets, fuzzy rules and fuzzy inference is provided in this paper. Membership functions and its type, defuzzification techniques, inference processes in fuzzy logic are covered in this paper.

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